THE IMPACT OF DIFFERENT VIRTUAL MANIPULATIVE TYPES ON CLASSROOM MATHEMATICAL DISCOURSE

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This study examined the influence of different virtual manipulative types on the nature of students’ discourse related to generalizing and justifying mathematical concepts. During 27 episodes, students worked on mathematics tasks using three different virtual manipulative types: linked, pictorial, and tutorial. The level of students’ discourse in generalization and justification was coded and analyzed for each episode and compared across virtual manipulative types. A one-way ANOVA indicated statistically significant differences in the quality of generalizations and justifications among the different virtual manipulative types. Other patterns indicate that certain virtual manipulative types may be more suited than others for encouraging meaningful mathematical discourse. The patterns and trends identified in this study contribute to the existing literature on the complex issues that surround mathematical discourse and the use of technology in the classroom.

PURPOSE

The purpose of this research study was to describe and categorize the nature of students’ mathematical discourse as they worked with various virtual manipulative types. A virtual manipulative (VM) is defined as “an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge” (Moyer, Bolyard, & Spikell, 2002, p. 373). As the use of technology in mathematics instruction becomes ubiquitous, questions arise regarding the role of different VM types in students’ learning experiences—particularly in the ways that students interact with each other and discuss mathematical ideas (Gray, Thomas, & Lewis, 2010; NCTM, 2007, 2014). The larger study from which this paper is taken employed a mixed methods case study design utilizing both qualitative and quantitative methods to analyze students’ mathematical discussions. The quantitative results provide the focus of this paper. Full qualitative results are described in other publications (see Anderson-Pence, 2014).

THEORETICAL FRAMEWORK

Virtual Manipulatives

With the advancement of computer capabilities, VMs have emerged as cognitive technology tools for use in mathematics classrooms. VMs provide teachers and students with expanded tools for thinking about mathematics concepts. Overall, research indicates that VMs positively contribute to students’ learning of mathematics concepts, and that design elements of VMs provide various affordances for students’ mathematical learning (e.g., Bolyard & Moyer-Packenham, 2012; Lee & Tan, 2014; Mendiburo & Hasselbring, 2011; Moyer-Packenham et al., 2014; Moyer-Packenham et al., 2013; Moyer-Packenham et al., 2015; Reimer & Moyer, 2005; Suh & Moyer-Packenham, 2008; Suh, Moyer, & Heo, 2005).
VM tools vary in the type of feedback they provide and the type of mathematical representation included (Bolyard & Moyer, 2007). For example, linked VMs are open-ended and present multiple representations of mathematical concepts that change simultaneously as they are manipulated. Linked VMs reflect the user’s actions and choices without dictating solution strategies. Pictorial VMs are visual representations of mathematics concepts similar to physical manipulatives. Like the linked VMs, pictorial VMs reflect the user’s actions and choices, but they do not include numeric symbols associated with the visual representation (Clark & Paivio, 1991; Paivio, 2007; Sfard, 1991; Zbiek, Heid, Blume, & Dick, 2007). Tutorial VMs are structured instructional tools designed to guide students to a conceptual or procedural understanding of mathematics. Tutorial VMs dictate specific solution strategies give direct feedback to users based on their adherence to those strategies.

Mathematical Discourse

Students develop understanding as they interact with others through verbal or nonverbal communications or written word (Vygotsky, 1978). Meaningful classroom discourse contributes to students’ understanding by promoting effective communication and articulation of thought (Piccolo Harbaugh, Carter, Capraro, & Capraro, 2008). One key aspect of small-group interaction is “co-construction,” defined by Mueller (2009) as a “form of collaboration in which an argument is built simultaneously by the learners from conception” (p. 141). Multiple studies have examined the process of mathematical explanation and reasoning (e.g., Carpenter, Fennema, & Franke, 1996; Hufferd-Ackles, Fuson, & Sherin, 2004). Notably, the framework for Robust Mathematical Discussion describes components of effective mathematical classroom discourse (Mendez, Sherin, & Louis, 2007). In this framework, robustness refers to the “mathematical and discursive strength of the discourse” (p. 42). Robust Mathematical Discussion categorizes students’ comments along two dimensions: mathematics and discussion. The mathematics dimension addresses three aspects of mathematical argumentation: representation, generalization, and justification. The discussion dimension examines three aspects of discourse: engagement, intensity, and building on others’ ideas. Discourse is most effective in promoting understanding when students’ discourse is ranked high in each of the Robust Mathematical Discussion dimensions.

Techno-Mathematical Discourse

To date, extensive research has been conducted on the nature of classroom mathematical discourse (e.g., Gee, 2005; Herbel-Eisenmann & Wagner, 2010; Iiskala, Vauras, Lehtinen, & Salonen, 2011; Imm & Stylianou, 2012; Nathan & Knuth, 2003; Wood & Kalinec, 2012). However, few studies exist on the interactions students have with each other when using technology to learn mathematics (e.g., Ares, Stroup, & Schaden, 2008; Evans, Feenstra, Ryon, & McNeill, 2011; Sinclair, 2005). The Techno-Mathematical Discourse (TMD) Framework (Anderson-Pence, 2014) was developed to more clearly define and analyze the interconnectedness of mathematical tasks, technology tools, and classroom discourse (see Figure 1).
METHODS

This study aimed to answer the following research question: How do different VM types influence the levels of generalization and justification in students’ mathematical discourse?

Participants

The study included 3 pairs of fifth-grade students ages 10–11 years (each pair consisting of one female and one male student). These students were selected based on their ability to verbally process their thinking and to get along with their assigned partner. Mathematics achievement was not a deciding factor when selecting students for this study. Classroom teachers assisted the researcher in selecting the students.

Procedures & Data Collection

Each pair of participating students shared a laptop computer while they interacted with nine different VMs: 3 linked, 3 pictorial, and 3 tutorial. Over four months, the 3 student pairs participated in 9 lessons using the VMs—a total of 27 episodes.

Data collection took place during 20–30-minute episodes as students worked together through assigned tasks. Two different video perspectives were recorded as data for further analysis. First, a face-capture perspective recorded the students’ mathematical discussions using the built-in camera located at the top and center of the computer screen. Second, a screen-capture perspective recorded what the students did with the VMs. This screen-capture included a record of mouse movement, mouse clicks, and external audio. Both perspectives were recorded simultaneously using Quicktime Player.

Analysis

The first stage of analysis focused on quantitizing the video data (Tashakkori & Teddlie, 2010). Speaking turns in each of the 27 episodes were transcribed and coded for levels of discourse according to the generalization and justification dimensions of the Robust Mathematical Discussion Framework (see Table 1). The number of codable speaking turns was tabulated to provide a measure of the quantity of discourse in each episode. Next, leveled codes were used to calculate composite scores—a measure of the quality of generalization and justification in each episode. Composite scores were calculated by a summation of the codes for each speaking turn within the episode divided by the total number of codable speaking turns, and multiplied by 100. For example, a discussion with 91 total speaking turns coded for justification—60 as statement (level 1), 13 as
explanation (level 2), and 18 as proof (level 3)—would yield a justification composite score of \[ \left( \frac{160 \times 1 + 15 \times 2 + 18 \times 3}{91} \right) \times 100 = 153.85. \] To determine reliability in the coding of each speaking turn, an additional researcher double-coded 10% of the video-recorded discussions. A comparison of the double codes resulted in 81% agreement (Creswell & Plano Clark, 2011).

<table>
<thead>
<tr>
<th>Level of Generalization</th>
<th>Level of Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>3  Generalization</td>
<td>3  Proof</td>
</tr>
<tr>
<td>2  Comparison</td>
<td>2  Explanation</td>
</tr>
<tr>
<td>1  Concrete</td>
<td>1  Statement</td>
</tr>
<tr>
<td>0  Not Codable</td>
<td>0  Not Codable</td>
</tr>
</tbody>
</table>

*Adapted from the Robust Mathematics Discussion Framework (Mendez et al., 2007)

Table 1: Video coding schema for levels of discourse

A one-way ANOVA on the amount of coded speaking turns per episode was conducted to compare the quantity of discourse for each VM type (i.e., linked, pictorial, and tutorial). One-way ANOVAs on composite scores were also conducted for generalization and for justification to compare the quality of discourse for each VM type.

In the final quantitative analysis, the data were examined for levels of discourse over the course of the students’ interactions. This analysis indicated differences in the progression of discussions among VM types. In order to compare the discourse progressions of discussions of varying lengths, each discussion was divided into quartiles according to the number of speaking turns. Then, for each quartile, the number of speaking turns coded for each level of generalization and justification was calculated.

RESULTS

A one-way ANOVA indicated no statistically significant differences among VM types in terms of the number of students’ speaking turns, \( F (1, 24) = 3.258, p = .056 \). However, one-way ANOVA comparisons of composite scores among the VM types did indicate significant overall differences in terms of the level of discourse (generalization: \( F (2, 24) = 9.460, p = 0.001 \); justification: \( F (2, 24) = 9.459, p = 0.001 \) with highest composite scores being associated with linked VMs across each measure.

A summary of the average number of speaking turns and composite scores for each VM type is presented in Table 2.

<table>
<thead>
<tr>
<th>Measure</th>
<th>Linked</th>
<th>Pictorial</th>
<th>Tutorial</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( M )</td>
<td>( SD )</td>
<td>( M )</td>
</tr>
<tr>
<td>Speaking Turns</td>
<td>98.67</td>
<td>39.60</td>
<td>111.22</td>
</tr>
<tr>
<td>Generalization</td>
<td>128.52</td>
<td>15.56</td>
<td>115.26</td>
</tr>
<tr>
<td>Justification</td>
<td>135.00</td>
<td>14.78</td>
<td>122.20</td>
</tr>
</tbody>
</table>

Table 2: Average speaking turns and composite scores for each VM type.
Quartile Analysis of Generalization Levels

For linked VMs, the highest level of generalization occurred steadily throughout the course of the discussions (see Figure 2). However, it occurred most frequently in the last quartile of the discussions. The second level of generalization—comparison—occurred in similar proportions in the first and second quartiles (14.10% and 14.20%), and then decreased for the third and fourth quartiles (1.71% and 5.26%). For pictorial VMs, the two highest levels of generalization occurred most during the last quartile. For tutorial VMs, discussion remained at the most basic level, concrete throughout the discussion. More statements were coded for the second level, comparison, in the first two quartiles of the discussions than for the last two quartiles. Speaking turns coded at the highest level accounted for less than 1% of the first and fourth quartiles with tutorial VMs.

Figure 2: Quartile analyses of generalization for all VM types

Quartile Analysis of Justification Levels

For linked VMs, levels of justification increased as the discussions progressed (see Figure 3). The percentage of speaking turns coded for explanation and for proof increased considerably after the first quartile (4.55% to 20.13% and 0.65% to 5.03%, respectively). For pictorial VMs, the levels of justification also increased as the discussions progressed, but not to the same extent as the linked VMs. The percentage of speaking turns coded for explanation and for proof increased after the first quartile (9.76% to 15.93% and 1.83% to 3.85%, respectively). For tutorial VMs, the most frequent occurrence of proof happened in the first quartile (2.52%). However, for the rest of the discussion, proof accounted for less than 1% of the speaking turns. The most frequent occurrence of explanation also happened in the first quartile (22.69%). Thereafter, the percentage of explanations dwindled to 10% or less for the remaining portion of the discussions.

Figure 3: Quartile analyses of justification for all VM types
In summary, the quartile analysis confirms that students’ discussions when using the tutorial VM type consistently reflected lower levels of generalization and justification. In addition, the analysis of discourse from the beginning to the end of each episode shows that when working with the linked and tutorial VM types, the level of generalization in students’ discussions remained constant. However, when working with the pictorial VM type, the level of generalization increased toward the end of students’ discussions. Levels of justification in students’ discussions remained relatively constant when working with the pictorial and tutorial VM types. However, when working with the linked VM types, levels of justification in students’ discussions increased after the first quarter of the episode.

EDUCATIONAL IMPORTANCE

Findings from this study suggest ways that teachers may effectively incorporate VM types into mathematics instruction to match students’ learning paths. First, pictorial and linked VMs may be more useful as students are developing their understanding of mathematics concepts. The flexibility of these VM types lends itself to an open exploration of mathematical ideas—guided either by the students themselves or by the teacher. Further, the linked VMs assist students in making connections between mathematics concepts and representations. In this study, students’ discussions when using this linked VMs typically reflected higher levels of generalization and justification. Through such robust discussion, students are more likely to learn mathematics in a meaningful way.

This study also suggests that the use of tutorial VMs may not be an effective instructional strategy for engaging students in mathematical discourse. Tutorial VMs are designed to walk an individual student through a concept at his or her own pace using focused feedback on performance. In this study, although the structured feedback included in the tutorial VM type effectively guided the students to a mathematical understanding, it did not encourage meaningful discussion between students. Students’ interaction with their partners was secondary to responding to the tutorials’ direct feedback. Due to the extremely structured nature of the tutorials, students did not feel the need to generalize or justify their answers with each other. Therefore, students’ discussions when using tutorial VMs typically reflected lower levels of generalization and justification.

LIMITATIONS

This study was designed as an exploratory study of the nature of students’ techno-mathematical discourse (TMD). At the time of this study, the construct of TMD was beginning to emerge and research on it was in initial stages. Therefore, the purpose of this research was to identify and characterize the construct (Lesh & Lovitts, 2000). The sample size was small and limited to 6 students. With so few participating students, the variations in students’ characteristics can have a profound effect on comparison results. Other factors that may have influenced the results of this study include students’ achievement, students’ familiarity with the VMs, and students’ perceptions of the VM types. However, these factors were beyond the scope of this study.

The broader classroom environment may also have been a contributing factor in the results of this study. Because the student pairs came from three different classrooms, it is possible that differences in classroom culture related to the sharing of ideas may have influenced how students interacted with each other in partner situations. The role of the teacher in influencing the nature of classroom
discourse was not addressed in this study. Future studies that address these factors will contribute to refining the TMD Framework and its generalizability.

**SCHOLARLY SIGNIFICANCE**

The patterns and trends identified in this study contribute to the existing literature on the complex issues that surround mathematical discourse and the use of technology in the classroom environment. More and more classrooms are using technology, and students are learning mathematics as they interact with the technology and with each other. However, we know very little about the interactions students have with each other when also interacting with technology to complete mathematical tasks. This study represents an intersection of the two research fields of VMs and classroom discourse and adds to the research literature on the impact of technology on classroom mathematical discourse.

**References**


Anderson-Pence, McGuire


